

AUTOMATION AND DATA PROCESSING IN AQUACULTURE

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APPLICATION OF LINEAR PROGRAMMING TO INDIVIDUAL FISH FARM PRODUCTION PLANNING

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Abstract. Decision making in a fish farm business presupposes a deep knowledge of the farming system itself and the market environment within which it operates. Assuming that the fish farm manager has all the data needed to control his business and to plan future production through a well established data recording system, he may then use further techniques which will give him high level information and guidelines for future policies.

Linear programming is a sophisticated, computer based technique which has recently become available to smaller businesses with the advent of affordable, but powerful microcomputers with simplified, 'friendly' software. It produces planning models which provide a decision making tool to indicate the way a farm should operate in order to achieve certain specific objectives. It is essential, for its use, that comprehensive records of the business should be available.

In this study the skills needed to use this technique, its data requirements, its strengths and usefulness under a fish farming environment were analysed. For this purpose "LPWYE" linear programming package running on an IBM PC microcomputer was applied to a fish farming business which operated the circular tank system to produce trout fingerlings for restocking.

The farming system was analysed and its data presented in the LP matrix form, which is the basic input for the technique. The production process and the marketing environment were faithfully reflected in the matrix and a solution was sought which aimed at a maximum profitability production plan, given the particular resource limits and market constraints of the farm.

From the study conclusions were drawn as to the usefulness and overall suitability of linear programming in a fish farming environment, and a case study is given as an example which states the basic requirements for LP, explains how the matrix can be constructed and how the results can be interpreted.

Keywords. Fish-farm planning; optimisation; linear programming; microcomputers; decision making.

INTRODUCTION

Planning future production is one of the essentials of (fish) farm management and decisions must be made as to which products, when and how much of them, are to be produced. Apart from these questions the technical processes which must be followed in order to achieve the end product, must be clearly defined. Therefore, production planning decisions must answer the following four basic questions:

- What to produce?
- When to sell?
- What quantity of each kind of product?
- How should a particular output be produced?

Possible feasible answers must comply with the existing resource/inputs on the farm and those which the farm is in a position to develop or acquire from outside. Moreover, these decisions must have an end result, a target. Management has to make decisions because resources are limited and therefore must be optimally combined to provide the desired outcome each time. The guides to a manager's decisions are data/information¹ and the techniques which process

¹Data is considered here as low-level, elemental, unprocessed, quantified representation of facts, whereas information results from the processing of data.

this information. The manager's personal judgement of risk and of uncertainties is then introduced and a decision made.

The implementation of both a data system, to collect and store data and information, and of processing techniques has been made possible for small firms with the use of electronic microcomputers. These machines are now affordable by small businessmen, like the average fish farmer, and provide sufficient computing power and ease of use. Highly sophisticated programmes for data and information processing are now accessible and 'friendly' and it is up to the individual manager to enhance his effectiveness using these management tools.

WHAT IS LINEAR PROGRAMMING

Linear programming is a mathematical planning technique based on matrix algebra and is best suited to a computer. It produces planning models which provide a decision making tool to indicate the way a farm should operate in order to achieve certain specific objectives. It is useful in fish farm planning because it determines the combination of production options (activities) and their producing techniques which will maximise revenue, and minimises the

production costs by the most efficient allocation of resources among the activities in a plan. The computer input must be arranged in an LP matrix which is simply a tabulated form of the farm's data required to identify the planning problem. In other words, the real life situation is expressed in a way that the LP programme can understand in order to work out the optimum solution.

USING LP ON THE INDIVIDUAL FISH FARM

The fish farm businessman will be concerned with decisions as to what quantities of which fish of what sizes should be produced at certain times and how much capacity his farm will require at various points in time in order to achieve his targets. Linear programming will choose from a series of options and provide an optimum solution based upon the current internal (his own farm's production affairs) and external (markets) situation. This suggested strategy can be checked and revised as circumstances change.

Using a microcomputer it is now possible to translate farm data into LP models. Realistic models can be set-up and the results can be best interpreted in terms of the particular farm's needs since the fish farmer can 'feel' and balance correctly what the computer suggests for the farm and formulate the computer input according to his own field experience.

Therefore, the system is planned for use by the fish farm manager himself with a minimum of interference from anyone external to his business environment, such as farm consultants or public advisory services.

SELECTION OF THE LP SOFTWARE

The microcomputer LP package which was used and is currently recommended through this study is 'LPWYE' developed at Wye College (University of London). It was developed for purely scientific and teaching purposes in an agricultural college by scientists with experience in farming. Therefore, is suited to agriculture and uses agricultural terminology. It was selected because it is easy to operate, very cheap, and, apart from the standard optimal solution, it provides a very useful amount of additional output. The maximum matrix size offered is appropriate for solving reasonable agricultural problems in practice. The package is well documented, fast in operation and available for the most popular microcomputers (CP/M and MS DOS operating systems).

FISH-FARM PLANNING ENVIRONMENTS

Because of the different management objectives which prevail in individual farms it is necessary to define two broad categories of fish farm planning environments:

1. Fish farm units which produce variable products, for example, farms producing fingerlings which may be sold at various sizes for restocking other farms, or farms which produce more than one aquatic species.
2. Fish farm units which produce a predetermined level and kind of product (or ideally due to market demand should standardise their output), for example, farms producing table-fish, smolt producers etc.

Within both of these two categories the fish farms may be distinguished according to their culture systems as follows:

- a) those farms where fish grading and their subsequent grouping according to size and growth potential is possible. In such cases fish growth can be monitored realistically and these are obviously farms operating tank or cage culture systems.
- b) those fish farms, operating mainly fish-pond systems where fish grading is extremely difficult and therefore not a practical proposition. The fish remain in the same production unit/pond from their very early stages of growth until harvest, and their growth is forecast according to the farm's past performance records.

Linear programming may be employed in all of the above cases since it is able to produce an ideal solution in every planning situation according to the specified management objectives, providing that the farm's environment is realistically represented in a suitable LP matrix.

AN EXAMPLE OF A FISH FARM PRODUCTION PLAN FORMULATION USING 'LPWYE'

'Trout Fisheries Ltd.' is an imaginary fish farm which operates on circular tanks and produces trout fingerlings for restocking other farms producing table fish. Imperial measurement units (used throughout this analysis) or metric may be used.

Farm Description

Tank capacities. The tanks on the farm were separated into groups by diameter; there were five tanks of 30ft diameter and twenty tanks of 12ft diameter. The water depth in these tanks was maintained constant throughout the year³ providing a total capacity of about 17,690 ft³ water all year round (a minimum water supply was assumed to be guaranteed by the local water authority, a fairly common situation).

Fish egg supplies and egg costs. The farm was able to produce a maximum of 1m eggs a year from its own brood stock, but could also import eggs from abroad, mainly from Denmark and the U.S. The anticipated egg input costs were as follows:

Own produced eggs	\$1.0 per '000,
Danish imported eggs	\$3.0 per '000,
US imported eggs	cost \$11.0 per '000.

Definition of the production options (activities)

The identification of all the possible final fingerling sizes which might be produced on the farm, should account for the different growth potential among the fish of a given population. The fingerlings came from either of three batches of eggs (ie. own eggs, eggs imported from Denmark and from the US) hatched on the farm at certain periods in the year. However, not all of the fish which started together continued to grow equally efficiently throughout, and as a result of this growth variation fish grading and putting fish of similar size together was practiced. As fish of similar growth potential tended eventually to be gathered together, they showed similar growth thereafter, thus, several different fish sizes might be produced at the same time.

Three distinct categories of fish, fast, average and slow growing were considered according to

their growth potential. These came from the same initial batch/origin and were eventually isolated through grading. Therefore, although a continuous fish-size distribution in the total population might be expected, consistent grading, apart from the well accepted benefit of improving the food conversion, grouped the fish by their growing efficiency and narrowed down the size deviation within each category. Thus, it was necessary to recognise three different fish sizes from each egg batch. This was taken into account in the possibilities under consideration for the production planning process.

Obviously, the relevant percentages of fast, average, and slow growing fish within the same batch should be revealed from the fish farm's historical records.

However, market supply and demand forces make quite clear to an alert fish farm manager the marketable sizes at which he should expect or prefer to sell his fingerlings. In this imaginary case, the market situation dictated that fingerlings could be expected to sell in combinations of the following sizes:

- small size 100/lb fish,
- average size 60/lb fish, and
- large size 16/lb fish.

The information about the fish growth together with the information about what the customers wanted, formed the production options which were open to the farm manager.

Estimation of food costs. For reasons of insufficient data from real situations, rather than for simplicity, the fish food conversion was assumed equal for all fingerlings irrespective of egg origin or timing of production cycle and even irrespective of the naturally occurring variability in growth potential among the fish of a certain batch which caused some fish to reach a specific size earlier than others. The food cost for each size of fish sold was calculated assuming a baseline FCR of 1.3 throughout the production cycle. Obviously, the food cost coefficient was influenced by the assumptions made about the FCR value.

The food costs of the various activities were calculated as follows:

- for 100/lb fingerlings	= \$ 5.304 per 1,000
- ... 60/lb ..	= \$ 7.278
- ... 16/lb ..	= \$22.208

Allowance for mortalities. The mortalities were assumed to be independent of the duration of production, the time it started, and of the final fish size produced, that is, as if all deaths occurred at the very initial stages and were not affected seasonally. In reality mortalities differed according to production timing, fish strain and duration of production cycle, but limited data records prevented this detail in being included in the matrix. The plan assumed that for a given end number of live fingerlings to be produced, a larger amount of eggs should be introduced at the start of the production cycle in order to offset the mortality losses throughout the process of rearing the fish.

In the LP matrix the cost of mortalities was taken into account by adjusting the egg cost value to allow sufficient eggs to guarantee a target number of fish after mortality had occurred. The fish farm records suggested that a 40.0% mortality rate for all origins of fish

could be expected, so, the egg costs for all three fingerling origins, allowing for the above mortality rate, were:

- 'Home' produced eggs, \$ 1.67 per '000,
- Danish imported eggs, \$ 5.00 per '000,
- US imported eggs, \$18.33 per '000 fingerlings.

Prices of the fish farm produce. No seasonal variation was assumed for the farm gate prices for the same size of fish of the same batch origin. The production plan assumed the following prices (excluding transport costs):

Fingerling size sold. Nos/lb	Farm gate price (\$)	
	Danish/USA	Own
100/lb	27.4	31.5
60/lb	30.8	35.4
16/lb	61.4	70.6

Transport costs were excluded from the model because no stable pattern could be assumed for them. In any case, the cost of deliveries would be charged to the customers, so there was no reason to allow for it in the LP matrix.

Fish farm labour restrictions. The workforce on the farm imposed some restrictions on the production plan since available labour was a relatively scarce resource. With the current labour force situation on the farm, accepted at least as far as the next production period was concerned, there was a limit on the biomass which could be sold/handled in any one month's overall activities. Moreover, there was a maximum level of fish sales from any one size group in each month, which was jointly determined by the market demands, the availability of transport, and the existing workforce capacity. So, some additional constraints to the plan were defined as follows:

- Maximum biomass limit of 15,000 lbs. to be sold/handled in any one month overall activities,
- Maximum level of sales in any one size group within each month set at 200,000 fish at 16/lb, 380,000 fish at 60/lb, and 480,000 fish at 100/lb.

At this point, with the assumptions, constraints and activities defined above, the LP matrix could be constructed; that is these parameters could be presented to the computer in a form which it was capable of processing.

The matrix which was finally constructed included twenty nine (29) alternative production options (activities) and thirty six (36) constraints. A small section of it is shown in Fig. 1, which reveals the arrangement of the parameters. The matrix appears as a rectangular area with rows and columns. Each column represents one 'activity' and each row represents one 'constraint'. The relationship between a constraint and an activity is expressed by the coefficient which has these as coordinates.

Revealing the Matrix Logic

An explanation of how the fish farm environment was represented in the matrix and some problems which may occur from time to time are described below.

The farm gate prices form the Net Revenue figures since the major variable costs were

accounted for separately in the matrix. Two major variable cost items were included:

- Egg purchasing or producing costs (OVACOST row and column),
- Food costs (FOODCOSTS row and column).

The farm was supposed to borrow from an unlimited capital source, say a friendly bank manager, which charged 13.5% per annum on the amount borrowed. This flexible assumption was built using the first two activities (Ovacost, Foodcost) which were combined with the first two constraint rows and bearing the same names to indicate clearly that they were dependent on each other. These constraints were used in the form of "tie lines". They were supplied with capital by the first two relevant activities. The 'supply' element was shown by the -1 (negative) value of the corresponding coefficients built into the body of the matrix. That is, for each unit (\$) of capital needed for a particular purpose, one unit of capital (\$) was supplied for this purpose. Moreover, these resource rows were charged 13.5%, that is, \$0.135 for each \$1 of capital supplied. This was shown by a negative (-1.135) Net Revenue value of these supplying activities which in fact represented cost of capital.

more than a unit ('000) of eggs was required for the production of one unit ('000) of fish. Specifically, in this model, with 40% mortalities, 1,670 eggs were needed to produce 1,000 fingerlings.

The calculation of the coefficients for the 'FOODCOST' row needed to take into consideration the food conversion efficiency of the fish. For example, for 1,000 fish of the 'OJUL100' activity (No 5) to be produced, \$5.304 for food was needed, which equals (weight of 1,000 fish ie. 101b) * (PCR ie. 1.3) * (cost of a unit of food weight ie. \$0.408/lb). Therefore, the 'FOODCOST' row coefficient for the above activity was 5.304.

The names given to the activities in the matrix may seem very strange, but these are created by the model user himself and must be informative and unambiguous whilst being no more than eight characters in length. For example, the name of activity No 4 (OJUL60) was derived from three pieces of information. The egg origin, the month of fish sale, and the size of the fish sold. So, O-JUL-60 is: - from Own eggs - sold in JULY - at 60/lb.

The title of the problem is: 'TROUT FISHERIES LTD' PRODUCTION PLAN OPTIMISATION

	1	2	3	4	5	6	7	8	9	10
	OVACOST	FOODCOST	OJUN100	OJUL60	OJUL100	OJUL16	OJUL60	OJUL100	OJUL16	OJUL60
Net Revenues	\$1	\$1	'000	'000	'000	'000	'000	'000	'000	'000
	-1.135	-1.135	31.500	35.400	31.500	70.600	35.400	31.500	70.600	70.600
1 OVACOST \$1	0.000		1.670	1.670	1.670	1.670	1.670	1.670	1.670	1.670
2 FOODCOST \$1	0.000	-1.000	5.304	7.278	5.304	22.208	7.278	5.304	22.208	22.208
3 JANWVOL ft3	17690.000					16.622				
4 FEBWVOL ft3	17690.000					16.622				
5 MARWVOL ft3	17690.000		0.562	0.562	0.562	16.622	0.562		0.562	0.562
6 APRWVOL ft3	17690.000		2.439	2.439	0.943	17.184	0.943	0.562	2.439	0.943
7 MAYWVOL ft3	17690.000		2.941	2.941	2.577	17.566	2.577	0.943	2.941	2.577
8 JUNWVOL ft3	17690.000		7.955	7.955	2.941	30.495	2.941	2.577	7.955	2.941
9 JULWVOL ft3	17690.000			16.622	7.955	39.068	7.955	2.941	16.622	7.955
10 AUGWVOL ft3	17690.000					7.955	16.622	7.955	27.917	16.622
11 SEPWVOL ft3	17690.000					9.944			36.127	27.917
12 OCTWVOL ft3	17690.000					12.401				36.127
13 NOVWVOL ft3	17690.000					16.622				
14 DECWVOL ft3	17690.000					16.622				
15 OEGGFAS '000	200.000		1.670	1.670					1.670	
16 OEGGAVR '000	500.000				1.670		1.670			1.670
17 OEGGSLW '000	300.000					1.670		1.670		
18 OEGGFAS '000	300.000									
19 OEGGAVR '000	750.000									
20 OEGGSLW '000	450.000									
21 UEGGFAS '000	200.000									
22 UEGGAVR '000	500.000									
23 UEGGSLW '000	300.000									
24 JANBIOM 1LB	15000.000									
25 FEBBIOM 1LB	15000.000									
26 MARBIOM 1LB	15000.000									
27 APRBIOM 1LB	15000.000									
28 MAYBIOM 1LB	15000.000									
29 JUNBIOM 1LB	15000.000									
30 JULBIOM 1LB	15000.000		10.000							
31 AUGBIOM 1LB	15000.000			16.667	10.000	62.500				
32 SEPBiom 1LB	15000.000						16.667	10.000		
33 OCTBIOM 1LB	15000.000								62.500	
34 NOVBIOM 1LB	15000.000									62.500
35 DECBiom 1LB	15000.000									
36 OOC16 '000	200.000									1.000

Fig. 1. The section of the LP matrix prepared for 'Trout Fisheries Ltd' showing the constraints and the first ten activities

The coefficients (positive) built into the body of the matrix represented how many units of a resource/constraint were needed for the production of one unit of an activity. For example, the calculation of the coefficients across the 'OVACOST' row embraced the assumed fish mortalities during the production of each activity. The cost of eggs required for the production of one unit ('000 fish) of an activity producing fish from a Danish egg batch was \$5.0, in spite of the normal cost of \$3.0/'000 eggs, because

The monthly 'WVOL' which stands for the Water Volume constraint restricted the plan to avoid exceeding the water amount available each month. The coefficients along each of these twelve rows expressed for every activity the monthly water requirements in water units (1 Ft³) of one unit ('000) of fish at their current stage of growth each time. For example, one unit of the activity No 3 ie. 'OJUN100' needed 2.439 units (Ft³) of 'MARWVOL'. So, 1,000 fingerlings, which came from own produced eggs and which were

intended to be sold in June at 100/lb., occupy 2.339 ft³ of water during March. These coefficients were calculated for each month based on data of the fish average size in that particular month and on the fish farmer's policy on stocking densities practiced for fish of that average size range. It is essential therefore that the growth pattern expected should be clear from the farm's data records. In essence, built into the LP matrix, was the monthly growth pattern of the fish and the policy on fish densities which depended on the different fish sizes. So, the monthly 'WVOL' coefficients expressed how much water space was needed by 1,000 fish of the given average size at the density stocked.

Since the production cycle of some activities extended to more than a year, some monthly coefficients were the compound of the water needs in the same month but for the successive years into which the production cycle extended.

The constraint rows numbered 15 to 23 all dealt with the fish egg resources of the farm. For each egg supply source (Own produced, Danish and U.S. imported) there were three constraint rows each representing a growth intensity group of fish expected from each batch of eggs, namely 'FAST', 'AVER', and 'SLOW' fish. The proportions of these fish-growth groups were indicated by the relative maximum limits set for them. These expected proportions, as the farm records showed, were 20% for 'Fast' fish, 50% for 'Average' fish and 30% for 'Slow' fish.

The final restraint was egg availability. The farm was able to produce 1m 'Own' eggs, to import 1.5m 'Danish' eggs and 1m 'U.S.' eggs. The upper limits of all of these restraints were those shown in the LP matrix.

The coefficients across these rows tied up only with the relevant activities, (eg. 'DEGGFAST' coefficients have been derived from only the 'Fast' Danish fish origin activities). These coefficients also depicted the assumed 40% mortality rate and for every unit ('000) of fish they revealed a need for 1.67 units ('000) of eggs ie. 1,670 eggs.

However, a very interesting point must be stressed here. Although it was possible to group the egg resources according to the future growth performance of the fish which would come from them, the computer was unable to 'understand' that these three growth categories of fish that had been defined should be produced jointly! that is 20 'fast' fish might be produced only if another 50 'average' and 30 'slow' were also produced. So, if the other resources on farm were suitable, the programme might very well produce an artificial optimal solution where there might be, say, only fast growing fish and the accepted proportions (in this case 20% fast, 50% average, and 30% slow fish) would be ignored.

Similar problems are also encountered in arable farming where crop rotations, that is, the proportions and sequences of various crops through the years must be planned. In arable farming, the end product of each cropping activity is standard (wheat grain, potato, etc.) and the production cycle is well established.

However, in fish farming, and in this model in particular, a set of different activities ('fast', 'average', 'slow') was defined, but these activities also had variable end products (different final fish sizes). So, even if the matrix

had been restricted in order to accept 50 'average' and 30 'slow' fish for every 20 'fast' ones produced, it would still not be possible in practice to instruct which of the several 'average' or 'slow' activities to combine with any specific 'fast' activity. Therefore, the techniques used when planning traditional farm production, such as the use of "proportional constraints", that is, constraint rows which give and receive 'permissions' for activities to be produced (Barnard and Nix, 1979), did not solve this particular problem. Another way would be to compound several activities of each category together but since it is possible in practice to include only a few of such combinations in the matrix, the planning flexibility, which is needed for a true optimum solution, is lost.

The above problem stemmed from a basic theoretical property of linear programming, that of additivity (or independence of the various activities in the matrix). This problem arises in cases where if one activity is introduced into the plan, then another must be brought in as well because the activities are not independent.

The solution to this problem, which did not allow the computer to 'corrupt' the optimum strategy, was to adjust the relevant amount of eggs of the different groups in the matrix in such a way that the computer simply accepted all offered eggs in whatever quantities they were offered and in the desired proportions between 'fast', 'average' and 'slow' fish that originated from them. This may often necessitate some preliminary runs of the programme and subsequent amendments of the egg resource maximum limits.

Regarding the finite capacity of the fish farm's workforce, constraints had been imposed on the maximum quantity of biomass to be sold or handled for sale each month. These monthly, 'BIOM' constraint rows were related to the activities by coefficients which were calculated as the amount of biomass of one unit ('000) of fish at the assumed final average size. Since each individual activity would produce fish ready for sale on one particular month only, it was obvious that it needed only be related to the particular biomass maximum constraint row which represented that particular month. For example, activity No. 5 produced 100/lb fish in July and therefore demanded 10/lbs out of the total handling capacity in that month ('JULBIOM' coefficient for activity 5 is 10.0).

Finally, as described in the introduction to the farm's planning environment, there were constraints on the total monthly amount of fish sold from each particular activity. Since to allow for this would necessitate the additional inclusion of as many more constraint rows as the number of activities in the matrix, and since most of them would eventually prove 'slack' - or non-participating in the plan-, it was found to be better to obtain a first provisional solution and then check it. If some activities were included at an excess level then constraints should be introduced for these only. Then the solution was checked again until there were no 'abuses' of the assumptions in it. In fact only one - the 36th - such constraint was required in the example case.

A complication may arise when planning a fish farm's production, especially on farms where grading of fish is possible and fish of similar sizes are brought together. There are defined production units - circular tanks - of

certain water capacities. When a number of fish of a specific size is transferred to a tank, then no more fish may be added to this tank unless they are of a similar size. Therefore, it is obvious that in such cases the space within a tank may be only partly utilised and the stocking densities of the fish lower than normal.

However, the LP programme assumes continuity of the water resource. Therefore, it may divide the water supply into arbitrary units which hold just the appropriate number of fish at the precise densities each time. Although the computer logic favours a very efficient water utilisation, this is sometimes unfeasible in practice especially for farms with a few large tanks. In such cases the computer produces an artefactual situation where different groups of fish may be allocated different portions of water within the same tank.

One answer to this problem is to include a monthly constraint regarding the total fish biomass that the farm as a whole is able to hold at any one time. Such a total monthly "MAXBIOM" constraint is related to all, fish-producing, activities and for all months. It was inevitable that when this constraint was introduced plenty of water seemed to remain unused, which was actually correct since the ideal stocking densities could not be followed consistently.

Another effective solution to the same problem would be to estimate the water which remained unused every month on farm due to the unavoidable inefficiencies of the stock management schedule and then subtract it from the monthly water available. This would cause the computer to calculate in terms of less water available and its optimum allocation among the fish groups would not create any further problems. With this latter approach a smaller LP matrix could be produced since the twelve extra constraint rows necessary to restrict the monthly maximum biomass would be avoided.

Interpretation of the LP Solution

The optimum plan. The selection of the best possible activities by the computer in terms of what, how much, and when to produce was based on the relative demands of the activities on the available resources (such as water, biomass handling capacity, etc.), their costs in terms of egg production or purchase, their relative mortality rates, their foodcosts, the farm gate prices they were sold at, and the resource's relative scarcity.

When the model described was run the optimum plan was as shown in Fig. 2.

Some rounding of the results should be made, since linear programming does not "think" in terms of integer activity levels. This happens because divisibility (or continuity) is a conceptual property of LP which assumes that resources and activities are divisible into infinitesimally small units.

In the above plan the suggested optimum was that 600,000 fingerlings should be produced from 1,000,000 eggs using own facilities allowing for 40% mortalities. From 1,500,000 eggs imported from Denmark 900,000 "Danish" fingerlings should be produced, and 1,000,000 imported eggs from the US would give 600,000 fingerlings both with a 40% mortality rate. Of all the above produced fish, 20% were expected to grow fast, 50% to grow at an average pace, and 30% to grow slowly.

If the fish origins were ignored and the sizes sold overall concentrated on, then, the farm was supposed to sell in total about 2,100,000 fingerlings.

49.0%	at the size of	16/lb,
26.3%	60/lb
24.7%	100/lb.

The total cost of produced and imported eggs was \$16,467.

The total food cost assuming an overall FCR of 1/1.3 was \$29,500.

The above figures bear a capital charge of 13.5%, that is, \$6,218 total capital cost for bank charges. The total Net Revenue achieved at farm gate prices was \$46,804 and, if we subtracted the 13.5% capital cost charges, the revenue figure would become \$53,022 which allowed for food and egg costs but not for the transport of fish.

Slack constraints. The monthly breakdown of the resources, which proved to be more than enough for the optimum plan, and the amount of their surplus, was given in the output under the heading of "Slack constraints" shown in Fig. 3.

The water surpluses were based on the initial assumption that a constant 17,690 ft³ water level was maintained on the farm throughout the year and a part of the surplus was certainly due to the imposed maximum biomass carrying-capacity in order to account for the inevitable inefficiencies of the stocking schedule.

Another resource in surplus was the monthly biomass handling and selling capacity, which was given a maximum limit of 15,000 lbs. Thus, it seemed that labour was not at all restricted, as far as handling fish for sale was concerned, and could be shifted elsewhere.

Activities not in optimal plan. The section titled "Activities not in optimal plan" in the 'LPWYE' output (Fig. 4) lists all those production possibilities which were not included in the optimum plan, and states the Net Revenue needed (in this case the farm gate prices) before these activities could be considered for inclusion, all other assumptions being equal.

When assessing the pricing system for the different products their relative prices and their demands on inputs must be considered. Since the pricing system used assumed equal prices for the same sizes of fish (even though they were produced during different periods of the year) the assessment of activities dealing with fish of the same size was based only on their relative resource requirements. For example, if it was necessary, for a particular product, to use a price which was dictated by market circumstances or personal intuition, then the model's information would show whether this price would be acceptable given the demands which the product made on the pool of resources compared with the return obtained from other product possibilities which competed for the same resources. This element of "opportunity cost" is clearly built into the logic of planning production using the linear programming technique.

Sensitivity analysis. In 'the plan' section of the output (Fig. 2) along with the activities in the optimal plan, there was additional, sensitivity analysis, that is information which stated the range of net revenues (farm gate

Maximum value of Net Revenue = 46803.803 - -

The Plan : N.R. range for which each activity level stays constant
 ***** (with the incoming variable)

	Level	Lower limit	Present N.R.	Upper limit
FOODCOST \$1	29590.560	-1.191(UEGGSLOW)	-1.135	-1.077(OJUL60)
OJUL16 '000	171.701	70.340(OJUL60)	70.800	71.653(OAUG60)
UMAY60 '000	106.898	29.640(UMAR100)	30.800	31.732(OAUG60)
OAUG100 '000	7.940	30.447(OAUG60)	31.500	31.760(OJUL60)
DSEP100 '000	200.683	27.192(OJUL60)	27.400	28.245(OAUG60)
DN0V60 '000	53.353	29.741(OAUG60)	30.800	31.425(OOCT16)
OSEP16 '000	89.824	66.773(OAUG60)	70.600	71.544(OJUL60)
OVACOST \$1	16467.066	-1.150(UEGGSLOW)	-1.135	0.000(OVACOST)
DAPR16 '000	179.641	53.586(DAUG100)	61.400	OPEN
OJUN100 '000	29.936	31.397(OJUL60)	31.500	35.327(OAUG60)
DFE860 '000	269.461	30.046(DJUL16)	30.800	OPEN
UMAR60 '000	119.760	30.472(UEGGFAST)	30.800	OPEN
UAUG16 '000	192.503	60.468(OAUG60)	61.400	80.161(OOCT60)
UMAY100 '000	179.641	27.120(UEGGSLOW)	27.400	OPEN
DJUN16 '000	195.066	57.232(OAUG60)	61.400	61.699(OJUL60)
OJUL100 '000	99.401	31.139(OAUG60)	31.500	33.786(OOCT16)
O0CT16 '000	200.000	68.314(OOCT16)	70.600	OPEN

Fig. 2. The optimal plan derived from the model

Slack constraints

	Lower limit	Surplus	Upper limit	Surplus
JANWVOL ft3			17690.000	2451.048
FEBWVOL ft3			17690.000	727.406
MARWVOL ft3			17690.000	1947.795
MAYWVOL ft3			17690.000	2955.074
APRWVOL ft3			17690.000	922.765
JULBIOM 1LB			15000.000	3274.682
NOVWVOL ft3			17690.000	4237.501
DECWVOL ft3			17690.000	2870.090
JUNBIOM 1LB			15000.000	2509.022
JANBIOM 1LB			15000.000	15000.000
FEBBIOM 1LB			15000.000	10508.892
MARBIOM 1LB			15000.000	13003.952
APRBIOM 1LB			15000.000	3772.455
MAYBIOM 1LB			15000.000	11421.926
AUGBIOM 1LB			15000.000	2889.143
SEPB10M 1LB			15000.000	7379.166
OCTBIOM 1LB			15000.000	2500.000
NOVBIOM 1LB			15000.000	14110.765
DECB10M 1LB			15000.000	15000.000

Fig. 3. The 'slack constraints' section of the LP output

Activities not in optimal plan *****			
		Present N.R.	N.R. needed before entry
OJUL60	'000	35.400	35.515
OAug60	'000	35.400	35.761
OOct60	'000	35.400	39.245
DAUG100	'000	27.400	35.214
DSEP60	'000	30.800	42.516
USEP16	'000	61.400	70.258
DOCT100	'000	27.400	28.560
DJUL16	'000	61.400	62.154
UNOV100	'000	27.400	28.560
UMAR100	'000	27.400	28.560
UJUL16	'000	61.400	62.154
UJUL60	'000	30.800	35.174

Fig. 4. Activities excluded from the plan

prices) within which the included activities would remain constant in the plan at the chosen levels. It would be worth changing the production plan only if the revenues were to go beyond these limits. This information on permissible net revenue changes was used to assess the stability of the optimum plan and large ranges indicate a much more 'confident' plan. Thus, the analysis of the activities included in the plan revealed the degree of their advisability when prices change, and, if the difference between the present net revenue figure and the lower limit of it was large, then it meant that the activity was relatively overpriced. On the other hand, in the 'Activities not in the optimal plan' section of the output (Fig. 4), the prices needed by the rejected activities in order to be considered for inclusion revealed by how much their current price needed to go up for them to be competitive with the others in the plan, i.e. that they were relatively underpriced.

The word "relatively" in the above analysis means that activities are over or underpriced relative to the others in the LP matrix in terms of their demands on scarce resources.

This analysis assumed that there was no correlation between any of the activities; if a price of one activity changed this would not cause changes in the price of other activities. In general the stability of one activity was assessed assuming that no other factor in the plan altered due to the change in the NR of the activity under examination.

In the printout of the plan (Fig. 2) beside the lower Net Revenue limit figure of an activity, the name of the new incoming activity which would replace it, or the name of the resource which would come into surplus when some units of the former activity were dropped out, was given in brackets. Moreover, in this same sensitivity analysis of the 'resistance' of an activity in the plan, the upper limit of the net revenue that each unit of the activity should earn for the plan before any more of this activity was included at the expense of some others, was also stated. In cases where the word 'OPEN' was stated instead of a figure, it meant that the plan used up the maximum amount allowed of that activity by the most constraining resource and the computer would have accepted more of it if it could. Beside the

upper limit figure of an activity the first candidate among the other activities, some units of which had to be sacrificed in order to allow the inclusion in the plan of more units of the former, was mentioned in brackets.

Binding constraints. Information was provided under the output section of "Binding constraints" concerning all those resources which were completely utilised in the plan and which moreover proved to be scarce, thus dictating the computer's 'decision' on the final form of the plan.

An important attribute of a binding constraint is the concept of its Marginal Value Product (MVP) which is defined as the amount of extra net revenue which is added to the final (optimum) solution for each extra unit of binding restraint that is made available.

According to the definition of MVP those constraints with a positive value reveal that the programme would preferentially accept more than the level imposed if more were available. It would then produce a final net revenue figure, increased by the amount of the MVP for every extra unit of scarce resource which would be made available. Constraints with negative MVP values for every extra unitary increase on their imposed level, would bring the total NR figure down by the amount of their MVP.

Therefore, when imposing on the plan more of a factor with a negative MVP, LP warns that the solution shifts away from the optimum. The opposite applies when the computer is free to select greater levels of those factors which bear a positive MVP.

The "Binding constraints" section, shown in Fig. 5 offered the MVP values for each exhausted resource and a sensitivity analysis which stated the lower and upper limits for the level of the resource that confined the availability of it. Throughout this range the MVP was constant. Beside the figures of the lower or upper limits, in parentheses, were the names of those resources which would go out of surplus or those activities which would be first reduced when these limits were exceeded. However, it should be born in mind that all other factors in the plan should be assumed to be independent and remain constant.

The market environment. Although a plan produced by linear programming may look totally feasible in terms of resources available and of management attitudes, the question of its applicability lies mostly on the influences from the marketing environment. This factor is external to the fish farm itself, but contributes drastically towards the formation of the production plan. For the present example study it was assumed that there were no such market influences and the plan has been formed assuming equal opportunities for all the products (activities).

However, if any market trends existed, which favoured one or the other fingerling size and/or timing, or if customers' preferences concentrated on one particular product, then this should be accounted for by the farm's management. The LP matrix should depict such external impositions and allow for them in the form of constraint limits (minimum, maximum or even equality constraints), which would contribute towards the formation of an optimum plan suitable for the real

market situation. In this discussion the fish farming business that was modelled was insufficiently large to be able to influence or regulate the market. If this assumption did not hold then it would be economically beneficial to impose its plan on the customers rather than to conform to their preferences. For example, a hatchery supplying the market with fingerlings at times when there is virtually no competition in a specific locality could preferably sell those fish sizes which are shown by, linear programming, to be the most profitable to the business.

SUITABILITY OF LP WHEN PLANNING FISH FARM PRODUCTION

When fish farm production is planned the main concern is with the strategic planning and control of production rather than the analysis of the technical efficiency of production operations. Linear programming is not supposed to show the way in which certain tasks, such as determining the correct feeding levels the fish farmer should use, should be carried out. It accepts the current production operations and their level of

Binding constraints *****		Resource supply range over which the M.V.P. is constant (with the outgoing variable)			
	M.V.P. \$/unit	Lower limit	Present Level	Upper limit	
OEGGSLOW '000	12.772	285.500(OAUG100)	300.000	1323.122(JULBIOM)	
OVACOST \$1	1.135	OPEN	0.000	16467.066(OVACOST)	
DEGGAVER '000	12.948	437.512(OAUG100)	500.000	1213.282(JUNBIOM)	
OOC16 '000	2.286	164.317(OJUN100)	200.000	214.575(DNOV60)	
FOODCOST \$1	1.135	OPEN	0.000	29590.561(FOODCOST)	
JUNWVOL ft3	0.378	10642.857(DJUN16)	17690.000	19140.295(JUNBIOM)	
DEGGFAST '000	12.322	150.006(OJUN100)	200.000	1679.413(DJUN16)	
DEGGAVER '000	6.963	513.029(OJUN100)	750.000	1461.030(OSEP16)	
OCTWVOL ft3	0.117	17159.457(DNOV60)	17690.000	19685.592(DSEP100)	
DEGGSLOW '000	8.611	46.495(OSEP100)	450.000	547.267(FEBWVOL)	
DEGGFAST '000	13.212	159.960(OJUN100)	300.000	343.249(APRWVOL)	
AUGWVOL ft3	0.122	17304.801(OAUG100)	17690.000	19428.231(AUGBIOM)	
UEGGFAST '000	0.197	0.000(UMAR60)	200.000	325.626(FEBWVOL)	
JULWVOL ft3	0.107	15869.281(JUNBIOM)	17690.000	17987.662(OAUG100)	
UEGGSLOW '000	0.168	0.000(UMAY100)	300.000	818.545(FEBWVOL)	
UEGGAVER '000	0.836	319.388(UMAY60)	500.000	656.486(APRWVOL)	
SEPWVOL ft3	0.408	15509.214(AUGBIOM)	17690.000	18812.307(OJUN100)	

Fig. 5. The 'Binding constraints' section of the 'LPWYE' printed output

However, competition is, more usually, a major factor and in these cases the computer's freedom of choice must be limited by imposing decisions in the LP matrix by defining certain production constraints (equalities) or giving them minimum acceptable values.

The essential difference between an equality and a minimum (or a maximum) constraint is that 'equalities' do not allow any sort of flexibility to the programme. They impose exactly the level of the resource entered. Minimum constraints however, allow the programme to exceed the level if it 'thinks' it is of benefit, but forbid any reductions to the set level. In fact the matrix may be reformed in order to accept as facts whatever product restrictions the marketing environment dictates. What the computer will then do is to find the optimum strategy under these restrictions. LP will also give the best possible (least costly) resource distribution for optimum results and a clear indication of the implications that the imposed strategies have upon profitability (MVP values). Such a planning model, where part of the activity levels are fixed while others are optimised, is referred to as a partial optimisation model. In no case would a partially optimised plan have a higher value in the solution than one, virtually the same, but without some of the production constraints, that had been fully optimised. This is made clear by the MVP values of the various constraints, which show the direction of the adjustments needed in the plan in order to move towards a 'full' optimum.

efficiency given in the form of the input-output coefficients which are built into the body of the LP matrix. However, if the data exists, LP may be used to check the impact of alternative production techniques and pinpoint possible technical inefficiencies of operations. The LP solution will reveal the relative economic efficiency of different production methods by selecting, in the optimal plan, the most profitable of them. The requirement is that information on the alternative production methods for the same output should be built into the model and form distinct activities in the matrix.

LP assists in the decision of the best "product mix" according to some clear idea of the business's primary objectives which in turn determine the objectives for production. The clear definition of the business's targets reflects the internal characteristics of the firm and its expected response to the conditions of its environment. Hence it influences the construction of the model and the interpretation of its results. The actual times within a year that a production plan should be generated depends on how stable the operating environment is in terms of internal processes and external influences, that might influence the optimum plan. Forecasts or predictions of future events, like the overall growth in the market, monthly demand of each product etc., are vital when planning future production. LP provides a solution which is subject to subsequent revisions according to the changing circumstances. Any solutions must be interpreted in the broad sense

of providing guidance towards making the best possible decisions.

The way LP was used in this study is orientated towards improving the resource allocation and planning of existing fish production activities rather than radically restructuring future production plans. To leave the latter possibility open to the model, data about new activities never previously attempted on the farm, must be found. If such data is generated by forecasting models, the reliability of which is questionable, the LP results will be equally doubtful. Data of this kind is thought to be better obtained either from other farms which already operate these activities and are willing to disclose information, or from experimental agricultural stations and consulting bodies.

In the case study a complete production cycle was reflected in the matrix by a monthly breakdown of resource requirements and production scheduling. It would be a further task for the manager to consider the detail of day-to-day running of the fish farm in pursuit of the optimal plan. Such an attempt frequently highlights the inadequacy of record-keeping and it is this identification of data problems that helps to improve the existing recording system.

The demands for detailed and accurate data appear to be but are definitely not excessive. In fact the same information is needed - although in a much more informal way - for any other less sophisticated farm planning technique, or even when the whole business management and decision making is based solely on the farmer's experience. The only serious demand of LP is the accurate statement of the problem. If a manager is so uninformed and unable to specify his scarce resources or the likely input-output relationships, his planning efforts, no matter which method is used, cannot be surrounded by much confidence.

Linear programming can also stand criticisms regarding some basic theoretical and structural concepts around which it has been developed. It might be criticised on the grounds of its theoretical inadequacy in representing realistically a 'real world' situation. In fact LP provides relatively short-term static solutions which assume perfect knowledge of all the values which take part in the matrix, although in actual life managing a fish farming system is a dynamic process surrounded by uncertainty. There are also the structural criticisms relating to LP's assumptions of linearity and continuity. Linear programming cannot handle adequately situations where integer quantities only are acceptable and as a matter of fact, linear relationships are rare in the real world.

But although it deals with straight line factor-product functions, taking successive discrete linear segments on these functions may approximate continuous curves. For example a matrix may reflect the monthly fish growth in the form of successive 'steps'. Fractional quantities in the final plan, wherever inappropriate, may be rounded up to the closest integer amount, and since no-one may ever be sure of his expectations, sensitivity analyses may be used extensively to show how possible changes could affect profitability. It is possible to formulate a set of standard solutions for different states of nature, thus, 'good', 'medium' and 'bad' plans may be generated and then some

subjective probabilities attached to them. Then it is up to the personal appreciation of risk to act along the lines of the one or the other alternative plan. A set of "security constraints" might be included in the matrix, that is impose a certain conservative idea on the final plan in order to provide for the worse case. Alternatively, the same conservative effect on the plan can be achieved by restricting the resources in the model to an excessively low level. For a formal evaluation of risk, complementary Risk Evaluation Models (REMs) can be used (M.C. Murphy, 1971), but this presupposes a sound data base which enables the production of statistical information about the probability distribution of the planning parameters such as growth rates, mortalities, prices, costs, etc. With such information available, a LP plan may be obtained first using mean expected values and then the solution tested for variation by means of a REM. If the degree of variation is unacceptable the solution can be modified to give lower variation/risk by altering the planning constraints and coefficients generally at the cost of providing lower total net revenue.

Although LP provides the optimum planning solution for every differently defined planning environment, it should be presumed that the entrepreneur aims always at maximising his profits since maximum profits need not in all cases maximise satisfaction. As far as no assumption is made about the individual's psychology or behaviour, it is difficult to decide whether a maximum profit plan as such is the fish farmer's objective. Since material demands are satiable and also because leisure is an essential ingredient of a good life, not everyone will be willing to put every resource at hand - and entrepreneurial time is one of the most important - into the service of obtaining maximum revenue. However, it would be sensible to accept that after allowing for the individual's personal pursuits, whatever resources and effort he is willing to put in the business, are optimised.

Thus, LP solves a strategic problem which is defined according to a planning environment which perfectly suits the fish farmer's style and business objectives. The solution is formed by the amount of resources that he defines as appropriate for his purpose, and by certain quantified conditions which must hold before any plan is put forward, such as a minimum amount of personal expenditure for leisure.

In summary then, the concept of the fish farm production planning system given here incorporates the following parts:

Firstly, the data system supplies the necessary data for the construction of the LP matrix. A well organised data system which gathers, analyses, stores and retrieves, information must precede all serious planning efforts. As far as LP is concerned, the data system will provide the manager with the following sets of data for matrix construction:

- the production process,
- resource availability,
- pricing and costing,
- alternative production possibilities,
- sales and marketing.

Secondly, it comprises the linear programming system. This is divided into the hardware, i.e.

the microcomputer system that the LP problem will be run on, and the software, i.e. the LP package which is to be employed for the solution. A third factor needed may be named "the human interface" and incorporates the three basic functions that the human operator should perform. Namely, maintain comprehensive, accurate records, construct a linear programming matrix and interpret the results. According to our research observations, these functions demand the manager's commitment to master the technique and are continuously sharpened through experience and personal involvement. Business orientated personalities with entrepreneurial skills will have the patience and be determined to devote personal effort and the time needed for such self-training now that suitable microcomputer software has come to the market. LP matrices will be small initially growing bigger with time along with the experience gained by the operator, since they may be easily extended to cater for more factors or amended as better data is obtained from an evolving data system. This latter factor of better data organisation will evolve in parallel with the competence of the manager since he will be subconsciously motivated to scrutinise his data figures and the overall trading situation of his farm. When examining a LP solution the manager's knowledge of his farming system and his expectations from it, formed over the years, will help him recognise if one or more restrictions are forgotten in the original formulation of the problem and as a result an unacceptable plan is produced. He will be in no difficulty to identify immediately a solution which is inappropriate for his farming system, return to the matrix which was responsible for the result and put it right.

CONCLUSIONS

In conclusion LP when used in fish farm production planning:

- stresses the paramount importance of the existence of a sound data recording system on the farm,
- points out any weaknesses of the existing data recording system because of its demands for consistent and verified data to be built into the matrix,
- shows the status of the various resources on the farm, the relative scarcity of each particular one and their value to the system,
- points out all relative advantages and disadvantages of the various production alternatives given the farm's resource status,
- will give guidelines as to modifying production processes, introducing new production possibilities, or introducing extra units of necessary inputs if this is to the overall benefit,
- provides sensitivity analyses of the stability of the proposed plan since the assumptions made about quantities, costs and prices may change,
- challenges the fish farmer to obtain a better understanding of his system and explore it further to reveal all of its potential for exploiting new market opportunities.

Finally, it is important to repeat that LP does not provide solutions to be followed as such. What it does is to provide guidelines for the best policy according to the current operational

environment which is reflected in the matrix. It should be considered as a valuable decision making tool, which in addition motivates a better organisation of farm data and deeper understanding of the fish farm system as a whole. It is hoped that it will become more familiar to the fish farm managers as a result of the continuing upward trend in the use of microcomputers.

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